Computation of Radiation Spectra in Compton Sources

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18 September 2017
Team Effort

Current Team
• Geoff Krafft (JLab, ODU, CAS)
• Erik Johnson (ODU, graduate student)
• Nalin Ranjan (high school)
• Aaron Brown (ODU, undergraduate student)

Past Members
• Kirsten Deitrick (ODU, JLab)
• Alicia Hofler (JLab, ODU)
• Cody Reeves (Northwestern University)

New Collaboration
• INFN Milano Team: V. Petrillo, I. Derbot, L. Serafini, C. Maroli…
Outline of the Talk

• Overview of Thomson/Compton scattering
  • Regimes and the big picture

• Frequency modulation (chirping) in Compton sources
  • Definition, origin, derivation
  • Practical realization
  • Improvement in bandwidth and photon yield
  • Improvement in higher harmonics

• Big picture: Computation of Compton spectra
  • What we can do now and what we want to do
  • The importance of theory/experiment collaboration
Compton Scattering

- When a relativistic electron beam interacts with a high-field laser beam, intense radiation is generated through Compton scattering.

\[ E_{\text{radiation}} = \gamma^2 (1 + \beta)^2 E_{\text{laser}} \approx 4\gamma^2 E_{\text{laser}} \]

Backscattering most interesting \((\Phi = \pi)\):

- Factor of \(4\gamma^2\) increase in energy

Krafft & Priebe 2010 (Rev. Acc. Sci. & Tech. 3, 147)
Thomson/Compton Scattering: The Big Picture

\[ \alpha_0 : \text{amplitude of the normalized vector potential } A \]
Thomson Scattering in Non-Linear Regime

- Region III: Non-linear Thomson regime
  - High laser intensity, low electron beam energy regime
  - Electron recoil is neglected
- Non-linearity destroys the quality of the scattered radiation
- Restore narrow-band property by chirping the laser pulse

Ghebregziabher, Shadwick & Umstadter 2013 (PR STAB 16, 030705)
The Origin of Laser Chirping

• **Brau 2004 (PR STAB 7, 020701):** (fixed-frequency laser pulse)
  • Computed spectrum for backscattered non-linear Thomson scattering
    *Specific solution:* backscattering (otherwise the same as in Krafft 2004)
  • Focused only on the fundamental peak
  • Uses *stationary phase method* to obtain approximate solutions

• **Ghebregziabher, Shadwick & Umstadter 2013 (PR STAB 16, 030705)**
  • *Great idea:* A frequency modulation of the laser pulse can lead to
    narrowing of the peaks in the spectrum of backscattered radiation
    *Empirical approximation:* not general, nor derived from first principles

• **Terzić, Deitrick, Hofler & Krafft 2014 (PRL 112, 074801)**
  • Derived an optimal, general frequency modulation (chirp)
  • Demonstrated that *chirping can perfectly recover the spectral width*
Optimal Chirping Prescription

- Krafft 2004 (PRL 92, 204802): (fixed-frequency laser pulse)
- Computed spectrum for backscattered non-linear Thomson scattering
- **General solution**: arbitrary angles, entire spectrum

\[
\frac{dE_\sigma}{d\omega d\Omega} = \frac{e^2 \omega^2 |D_x(\omega)|^2}{8\pi^2 c^3} \sin^2 \phi,
\]

\[
D_x(\omega) = \int \frac{d\xi}{\gamma(1+\beta_z)} \frac{eA_x(\xi)}{mc^2} \times \exp\left[i\omega\left(\frac{\xi(1-\beta_z \cos \theta)}{c(1+\beta_z)}\right)\sin \theta \cos \phi\right] \frac{eA_x(\xi)}{c\gamma(1+\beta_z)} \int_{-\infty}^{\xi} \frac{eA_x(\xi')}{mc^2} d\xi' + \frac{(1+\cos \theta)}{c\gamma^2(1+\beta_z)^2} \int_{-\infty}^{\xi} \frac{e^2 A_x^2(\xi')}{2mc^4} d\xi'.
\]

Laser pulse: \( \tilde{a}(\xi) = \frac{eA_x(\xi)}{mc^2} = a(\xi) \cos \left(\frac{2\pi}{\lambda_0} \xi\right) \)

- **Non-linear terms**
  - High-field non-linearities induce *ponderomotive* broadening of the spectrum

Effect. motion spec. Krafft 2004, Fig. 2:
- low field (linear)
- high field (non-linear)
Optimal Chirping Prescription

- Introduce *frequency modulation* \( f(\xi) \) of the laser pulse in Krafft 2004:

\[
\frac{dE_\sigma}{d\omega d\Omega} = \frac{e^2 \omega^2 |D_x(\omega)|^2}{8 \pi^2 c^3} \sin^2 \phi,
\]

\[
D_x(\omega) = \int \frac{d\xi}{\gamma(1 + \beta_z)} \frac{eA_x(\xi)}{mc^2} \times \exp \left[ i \omega \left( \frac{\xi(1 - \beta_z \cos \theta)}{c(1 + \beta_z)} \right) - \frac{\sin \theta \cos \phi}{c\gamma(1 + \beta_z)} \int_{-\infty}^{\xi} \frac{eA_x(\xi')}{mc^2} d\xi' + \frac{(1 + \cos \theta)}{c \gamma^2 (1 + \beta_z)^2} \int_{-\infty}^{\xi} \frac{e^2 A_x^2(\xi')}{2 \gamma^2 c^4} d\xi' \right].
\]

Laser pulse:

\[
\tilde{a}(\xi) \equiv \frac{eA_x(\xi)}{mc^2} = a(\xi) \cos \left( \frac{2\pi}{\lambda_0} \xi f(\xi) \right)
\]

Krafft 2004: \( f(\xi) = 1 \)
Now: allow \( f(\xi) \) to vary

- Normalization for the frequency modulation function \( f(0)=1 \)
  - Maximum frequency at the highest field (\( \xi=0 \))

- The problem: find the chirping prescription (modulation function) \( f(\xi) \) which optimally restores the narrowband property of the \( D_x(\omega) \)
Optimal Chirping Prescription

- Terzić, Deitrick, Hofler & Krafft 2014 (PRL 112, 074801)

\[ \frac{dE_\sigma}{d\omega d\Omega} = \frac{e^2 \omega^2 |D_x(\omega)|^2}{8\pi^2 c^3} \sin^2 \phi, \]

\[ D_x(\omega) = \int \frac{d\xi}{\gamma(1 + \beta_z)} \tilde{a}(\xi) \exp \left[ -i\omega \left( \frac{1 - \beta_z}{c(1 + \beta_z)} \xi + \frac{1}{c\gamma^2(1 + \beta_z)^2} \int_{-\infty}^{\xi} \tilde{a}^2(\xi') d\xi' \right) \right] \]

\[ = 0 \quad \text{(Non-linear terms)} \]

FFT of the laser pulse

Laser pulse:

\[ \tilde{a}(\xi) = \frac{eA_x(\xi)}{mc^2} = a(\xi) \cos \left( \frac{2\pi}{\lambda_0} \xi \right) \]

Dx(\omega):

\[ A_x(\xi): \text{transverse comp. of vector potential} \]

For backscattered case \((\theta=0)\) and with

\[ \frac{1}{\gamma^2(1 + \beta_z)^2} = \frac{1 - \beta_z}{1 + \beta_z}, \]

where

\[ \tilde{a}(\xi) \equiv eA_x(\xi)/(mc^2) = a(\xi) \cos \left( \frac{2\pi}{\lambda_0} \xi f(\xi) \right) \]
Optimal Chirping Prescription

• Substituting \( \cos \left( \frac{2\pi \xi f(\xi)}{\lambda_0} \right) = \frac{1}{2} \left( e^{2\pi i \xi f(\xi)/\lambda_0} - e^{-2\pi i \xi f(\xi)/\lambda_0} \right) \) we get

\[
D_x(\omega) = \frac{1}{2} \int \frac{d\xi}{\gamma(1 + \beta_z)} a(\xi) \exp \left[ -i \left( \omega \frac{1 - \beta_z}{c(1 + \beta_z)} \left( \xi + \int_{-\infty}^{\xi} \tilde{a}^2(\xi')d\xi' \right) - \frac{2\pi}{\lambda_0} \xi f(\xi) \right] \]

\[
= \frac{1}{2} \int \frac{d\xi}{\gamma(1 + \beta_z)} a(\xi) \exp \left[ i \left( \omega \frac{1 - \beta_z}{c(1 + \beta_z)} \left( \xi + \int_{-\infty}^{\xi} \tilde{a}^2(\xi')d\xi' \right) + \frac{2\pi}{\lambda_0} \xi f(\xi) \right] \]

• Phase: \( \Phi(\xi) \equiv \frac{\omega(1 - \beta_z)}{c(1 + \beta_z)} \left( \xi + \int_{-\infty}^{\xi} \tilde{a}^2(\xi')d\xi' \right) - \frac{2\pi}{\lambda_0} \xi f(\xi) \).

• Apply the stationary phase method:
  • When integrand is rapidly oscillating around 0, as in \( \sin(x) \) or \( \cos(x) \), it cancels itself everywhere except near
  • Second term vanishes because phase is monotonically increasing
  • Only the first term resonates

\[ \frac{d\Phi(\xi)}{d\xi} = 0 \]

\[ \text{Fig. 6.19 Plot of } \phi \text{ and } \cos \phi \text{ for } \phi = \pi(\xi^2 \cdots) \]
Optimal Chirping Prescription

- Near the resonant frequency $\omega = \omega_0/(1 + a_0^2/2)$, where
  
  \[ \omega_0 = (1 + \beta_z)^2 \gamma^2 2\pi c / \lambda_0 = \frac{1 + \beta_z}{1 - \beta_z} 2\pi c / \lambda_0 \]

  \[ \Phi(\xi) = \frac{2\pi}{\lambda_0} \left[ \frac{1}{1 + a_0^2/2} \left( \xi + \int_{-\infty}^{\xi} \bar{a}^2(\xi') d\xi' \right) - \xi f(\xi) \right] \]

- We can also approximate:
  \[ \int_{-\infty}^{\xi} \bar{a}^2(\xi') d\xi' = \frac{1}{2} \int_{-\infty}^{\xi} a^2(\xi') d\xi' \]

  \[ \Phi(\xi) = \frac{2\pi}{\lambda_0} \left[ \frac{1}{1 + a_0^2/2} \left( \xi + \frac{1}{2} \int_{-\infty}^{\xi} a^2(\xi') d\xi' \right) - \xi f(\xi) \right] \]

- To keep phase constant:
  \[ \frac{d\Phi(\xi)}{d\xi} = 0 \quad \implies \quad \frac{d}{d\xi} [\xi f(\xi)] = \frac{1 + a^2(\xi)/2}{1 + a_0^2/2} \]

- Finally, with BCs $f(0) = 1$, we obtain the general solution:
  \[ f(\xi) = \frac{1}{1 + a_0^2/2} \left( 1 + \frac{1}{2\xi} \int_{0}^{\xi} a^2(\xi') d\xi' \right) \]

*Terzić, Deitrick, Hofler & Krafft 2014 (PRL 112, 074801)*
Optimal Chirping Prescription

- So, how well does this exact frequency modulation function do?

\[ a_0 = 0.587 \]

Scattered radiation with \( f(\xi) = 1 \) (no frequency modulation)

**Physically minimal bandwidth** of the scattered radiation spectrum:

FFT of the laser pulse

Scattered radiation with \( f_{\text{exact}}(\xi) \) matches the FFT of the pulse

As good as it gets!

**Perfect restoration of the narrowband spectrum**

Restores all harmonics at the same time
Practical Realization of Laser Chirping

- Chirping the electron beam results in a chirped output laser pulse from an FEL oscillator

- How well can *judicious* RF chirping improve the spectrum?
- Compute first the modulation function due to RF chirping:

\[
\frac{d}{d\xi} [\xi f(\xi)] = \cos^2\left(\frac{2\pi \xi}{\lambda_{RF}}\right) \quad \rightarrow \quad f_{RF}(\xi) = \frac{1}{2} + \frac{1}{\xi} \frac{\lambda_{RF}}{8\pi} \sin\left(4\pi \frac{\xi}{\lambda_{RF}}\right)
\]
Efficiency of RF Laser Chirping

\[ a_0 = 0.587, \gamma = 100 \]

**Peak height**

**Substantial Returns**

**Close to perfect**

1st Harmonic: \( \lambda_{RF} = 394 \)

3rd Harmonic: \( \lambda_{RF} = 323 \)

5th Harmonic: \( \lambda_{RF} = 305 \)
From a Single Electron to a Distribution

- Terzić, Deitrick, Hofler & Krafft 2014 (PRL 112, 074801) work was for a single-electron Compton scattering:
  - Perfect narrowing of the backscattered spectra
  - Analytical solution for an arbitrary laser pulse

- Natural next questions to ask:
  - What about an electron beam with an energy distribution?
    - Does frequency chirping help here?
    - Can we derive spectra for scattered radiation?
  - Why is this important?
    - Higher harmonics
    - Applications: laser plasma accelerators

- The answers are reported in our recent publication
  Terzić, Reeves & Krafft 2016 (PR STAB, 19, 044403)
Chirping Vs. Electron Beam Energy Spread

• We developed a tool to study the interplay between the two components of the bandwidth: laser spread and e-beam spread.

\[ \lambda = 1 \, \mu m, \ a_0 = 0.707, \ Q = 100 \, pC, \ E_b = 163 \, MeV \]
Improved Photon Yield

Energy spread of the laser pulse dominates

Our new results allow us to quantify this intermediate region

Energy spread of the electron beam dominates

\[ \lambda = 1 \text{ \mu m} \]
\[ a_0 = 0.707 \]
\[ Q = 100 \text{ pC} \]
\[ E_b = 163 \text{ MeV} \]

Terzić, Reeves & Krafft 2016 (PR STAB, 19, 044403)
Improved Photon Yield in Fundamental Peak

Increase in photon yield with perfect chirping (1\textsuperscript{st} harmonic with vs. 1\textsuperscript{st} without chirping)

Laser-plasma accelerator (LPA) based Compton sources domain of operation (limited by non-linear ponderomotive broadening)

Presently: \(\sigma \geq 3\%\)
\(\sigma < 1\%\) “realistic” (Geddes et al. 2015, NIM B 350, 116)

Entering a regime where chirping makes huge improvements

\(\lambda = 1 \ \mu\text{m}\)
\(Q = 100 \ \text{pC}\)
\(E_b = 163 \ \text{MeV}\)
Improved Photon Yield in Higher Harmonics

Increase in photon yield with perfect chirping (3\textsuperscript{rd} harmonic with chirping vs. 1\textsuperscript{st} without)

0.1\% FWHM energy spread

Backscattering peaks

\[ E = \frac{4\gamma^2 E_p n}{1 + \frac{1}{2} a_0^2} \]

\( n \): order of harmonic
\( \gamma \): electron beam

\textit{Same energy:}
1\textsuperscript{st} harmonic with \( E \)
3\textsuperscript{rd} harmonic with \( E/\sqrt{3} \)

Using 3\textsuperscript{rd} harmonic \( \rightarrow \) \( \sqrt{3} \) savings in energy

\( \lambda = 1 \text{ \mu m}, \ Q = 100 \text{ pC}, \ E_3 = 163 \text{ MeV}, \ E_1 = 282 \text{ MeV} \)

\[ R_{13} \equiv \frac{(d^2 N/d\omega d\Omega)_{\text{FM,3}}}{(d^2 N/d\omega d\Omega)_{\text{nonFM,1}}} \]
The Big Picture: Modeling Compton Scattering

• Modeling the laser:
  • 0D: Constant laser field
  • 1D: Plane wave (pulsed along $z$-axis)
  • 3D: Laser pulse

• Modeling the electrons:
  • On axis (no angles)
  • Off axis (with angles)

• Most realistic description:
  • 3D laser pulse
  • Off axis electrons
    • realistic electron beam distribution with energy spread and emittance
The Big Picture

Electron Model: On Axis

Electron Model: With Angles

This is where we want to be
New Code for Computation of Compton Spectra

• First reported in Krafft et al. 2016, recently made more efficient
• For now only works in the linear Compton regime (low intensity)
• Direct computation of probability of scattering of each individual electron from an arbitrary distribution
  • Solves the statistical issues that plagues CAIN
• Can take in an arbitrary electron beam distribution
  • Including energy spread and emittance
• Fast: computes spectra for about 2000 particles per minute
• Allows for study and verification of bandwidth scaling laws
• We would like to generalize it into a comprehensive code:
  • Include 3D laser pulse model (as in Maroli et al. 2017)
  • High laser intensity (non-linear) regime
  • Include laser chirping
Conclusion: Why is Chirping Important?

• Chirping removes the non-linear ponderomotive broadening – the only obstacle to accessing the high-intensity regime
• Chirping can restore narrowband property of all the harmonics in the spectrum
• For electron beams with small to moderate energy spread (< 10% FWHM) such as those in LPA, chirping
  • perfectly restores the radiation bandwidth
  • significantly increases the photon yield
• A desired photon yield at the same energy can be achieved with a 1\textsuperscript{st} harmonic without chirping or 3\textsuperscript{rd} or 5\textsuperscript{th} harmonics with chirping given the high enough intensity (now accessible)
  • reduction in electron beam energy by $\sqrt{n}$
  • substantial savings in design, construction and operation cost
Importance of Theory/Experiment Collaboration

• Our theory needs experimental verification to give our ideas – chirping, use of higher harmonics – widespread acceptance

• Experimental verification would have transformative effects on:
  • Traditional Thomson and Compton sources
  • Laser plasma accelerator-based Compton sources
  • Design and cost of operation of future high-intensity Compton sources

• Theory/experiment collaboration could enable a new technology – low-cost, compact sources of intense x-rays with many uses:
  • Basic sciences: material design, chemistry, tomography
  • Applications to medicine, security, industry...
Papers

• Krafft 2004, PRL, 92, 204802
• Terzić, Deitrick, Hofler & Krafft 2014, PRL, 112, 074801
• Terzić & Krafft 2016, PR AB, 19, 098001
• Terzić, Reeves & Krafft 2016, PR STAB, 19, 044403
• Krafft, Johnson, Deitrick, Terzić, Kelmar, Hodges, Melnitchouk & Delayen 2016, PR AB 19, 121302
• Maroli, Petrillo, Drebot, Serafini, Terzić & Krafft 2017, submitted to PR AB
Backup Slides
Optimal Chirping Prescription

- Stationary phase method: *the gift that keeps on giving*

  Exact solution: the height of the peak of the radiation spectrum

  From before:
  \[
  D_x(\omega) = \frac{1}{2\gamma (1 + \beta_z)} \int_{-\infty}^{\infty} a(\xi) d\xi
  \]

  Gaussian envelope:
  \[
  a(\xi) = a_0 \exp\left(-\frac{\xi^2}{2(\sigma \lambda_0)^2}\right) \implies \int_{-\infty}^{\infty} a(\xi) d\xi = \sqrt{2\pi} \sigma \lambda_0 a_0
  \]

  \[
  \frac{|D_x(\omega)|}{\lambda_0} = \frac{\sqrt{2\pi} \sigma a_0}{2\gamma (1 + \beta_z)}
  \]

  Example from Krafft 2004:
  \[
  a_0 = 0.587, \sigma = 16.33, \gamma = 100,
  \]

  \[
  |D_x(\omega)| / \lambda_0 = 0.06.
  \]
Optimal Chirping Prescription

- Stationary phase method: *the gift that keeps on giving*

  Exact solution: number of subsidiary peaks in the non-modulated spectrum

Previous *empirical* estimate:
(Heinzl et al. 2010)

\[ N_{\tau} = 0.24 \, T [\text{fs}] a_0^2 \]

*(only for \( \lambda = 800 \text{ nm} \) and Gaussian envelope)*

Our *general, exact* solution:
(any envelope \( a \))

Gaussian envelope:

\[ N_{\tau} = \frac{\sqrt{2\pi c}}{4\lambda_0} Ta_0^2 \implies N_{\tau}(\lambda_0 = 800\text{nm}) = 0.235T[\text{fs}]a_0^2 \]

Importance of the new result: given the pulse duration \( T \), and the number of peaks \( N_{\tau} \), compute the field strength \( a_0 \).
Scattering off an Electron Beam

- Non-monochromatic beam $\rightarrow$ non-zero energy spread
- TDHK 2014 arrived at a fortuitous property
  - Perfect frequency modulation function is energy-independent
    \[ f(\xi) = \frac{1}{1 + \frac{a_0^2}{2}} \left( 1 + \frac{1}{2\xi} \int_{0}^{\xi} a^2(\xi') d\xi' \right) \]
  - Same perfect chirp should work for all electrons in a beam!
- Location of the peaks in the spectrum *does* depend on energy
  - Superposed spectrum is broadened (peaks in different places)
- The bandwidth of the scattered radiation has two components
  1. Electron beam energy spread
  2. Ponderomotive broadening of the individual electron spectra
- We can’t do anything about 1, but TDHK 2014 tells us that we can completely undo 2! How good is that?
Scattering off an Electron Beam

- Superposition of individual electron spectra from the beam

\[
\left( \frac{d^2 E(\omega)}{d\omega d\Omega} \right)_{\text{beam}} = \int_{1}^{\infty} N(\gamma) \frac{d^2 E(\gamma, \omega)}{d\omega d\Omega} d\gamma
\]

- Brute-force computation is numerically expensive!

- Use a clever mathematical trick
  - Electron energy $\gamma$ only shifts and scales the frequency content form $D_x$:

\[
\frac{d^2 E(\gamma, \omega)}{d\omega d\Omega} = \left( \frac{d^2 E(\gamma)}{d\omega d\Omega} \right)_n \left( \frac{\omega}{\omega_0(\gamma)} \right)^2 \left| \tilde{D}_x \left( \frac{\omega}{\omega_0(\gamma)} \right) \right|^2
\]

  - Only need to compute one frequency content form $D_x$ and use to compute the integral above
Scattering off an Electron Beam

- Computation of \[ \left( \frac{d^2 E(\omega)}{d\omega d\Omega} \right)_{\text{beam}} = \int_{1}^{\infty} N(\gamma) \frac{d^2 E(\gamma, \omega)}{d\omega d\Omega} d\gamma \]
can be carried out numerically in general
- Frequency modulation for a single electron “undoes” the nonlinear broadening of the spectrum and renders it analytical
- After some tedious mathematics, we can obtain an analytic approximation for the beam spectrum for a general laser pulse

![Single-electron spectrum](image1)

![Beam spectrum (34% FWHM energy spread)](image2)
Improvement in Higher Harmonics

Increase in photon yield with perfect chirping (3\textsuperscript{rd} harmonic with chirping vs. 1\textsuperscript{st} without)

LPA Compton sources

Backscattering peaks

\[ E = \frac{4\gamma^2 E_p n}{1 + \frac{1}{2}a_0^2} \]

\( n \): order of harmonic
\( \gamma \): electron beam

Same energy:
1\textsuperscript{st} harmonic with \( E \)
3\textsuperscript{rd} harmonic with \( E/\sqrt{3} \)

Using 3\textsuperscript{rd} harmonic \( \rightarrow \) \( \sqrt{3} \) savings in energy

\( \lambda = 1 \) \( \mu \)m, \( Q = 100 \) pC,
\( E_3 = 163 \) MeV, \( E_1 = 282 \) MeV
Using Higher Harmonics (5th)

Increase in photon yield with perfect chirping (5th harmonic with chirping vs. 1st without)

\[ \lambda = 1 \, \mu m, \quad Q = 100 \, pC, \]
\[ E_3 = 126 \, MeV, \quad E_1 = 282 \, MeV \]

\[ \sqrt{5} \text{ savings in energy} \]
ODU Compact Compton Source

- 500 MHz, 4.2 K accelerating section, comprising of:
  - SRF re-entrant gun
  - 4 double-spoke SRF cavities

**e-beam properties**

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<td>$\beta_y$</td>
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**X-ray source properties**

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<td>ph/(s-0.1%BW)</td>
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<td>in 0.1%BW</td>
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<tr>
<td>Average brilliance</td>
<td>$1.0 \times 10^{15}$</td>
<td>ph/(s-mm$^2$-mrad$^2$-0.1%BW)</td>
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</tbody>
</table>

Krafft et al. 2016 (PR AB 19, 121302)
ODU Compact Compton Source

- CST Microwave front-to-end simulation produced this e-beam:
• Our Compton code produced this spectrum from that distribution

FIG. 12. Number spectra for the Old Dominion University Compton source with 10 pC electron bunch charge. Left: For apertures $1/40\gamma$, $1/20\gamma$, and $3/20\gamma$, 4,000 particles were used in generating each curve. For aperture $1/10\gamma$, $4,875,600$ particles were used in generating the plot. Right: The same as the panel on the left, except on the log scale.
Practical Realization of Laser Chirping

- Here is what the harmonics look like

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1\textsuperscript{st} Harmonic: $\lambda_{RF} = 394$

3\textsuperscript{rd} Harmonic: $\lambda_{RF} = 323$

5\textsuperscript{th} Harmonic: $\lambda_{RF} = 305$

7\textsuperscript{th} Harmonic: $\lambda_{RF} = 295$

9\textsuperscript{th} Harmonic: $\lambda_{RF} = 299$

---

$\frac{dE}{d\omega d\Omega}$ [J/sr]

Scaled Frequency $\omega/\omega_0$

$\frac{dE}{d\omega d\Omega}$ [J/sr]

Scaled Frequency $\omega/\omega_0$

$\frac{dE}{d\omega d\Omega}$ [J/sr]

Scaled Frequency $\omega/\omega_0$

$\frac{dE}{d\omega d\Omega}$ [J/sr]

Scaled Frequency $\omega/\omega_0$

---

$f(\xi) = 1$

Exact $f(\xi)$

$f(\xi; \lambda_{RF})$

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September 18, 2017

Radiation Spectra in Compton Sources
Scaling of the Linewidth

$$\frac{\Delta E_{ph}}{E_{ph}} \sim \sqrt{\left[ \frac{\Psi^2}{\sqrt{12}} + \frac{\bar{P}^2}{1 + \sqrt{12}\bar{P}^2} \right]^2 + \left[ \frac{(2 + X)}{1 + X} \frac{\Delta \gamma}{\gamma} \right]^2 + \left[ \frac{1}{1 + X} \frac{\Delta E_L}{E_L} \right]^2 + \left( \frac{M^2 \lambda_0}{2\pi w_0} \right)^4 + \left( \frac{a_0^2/3}{1 + a_0^2/2} \right)^2}$$

aperture  emittance  e-beam e spr.  p-beam e spr.  ponderomotive broadening

Curatolo, Drebot, Petrillo & Serafini 2017, PR AB 20, 080701
FIG. 6. On-axis radiated energy density from the scattering of a 300-MeV electron by a 90-fs FWHM and $1.6 \times 10^{18}$-W/cm$^2$-peak-intensity chirped laser pulse (blue line) and transform-limited pulse (black line). The photon flux per unit solid angle is approximately $4.1 \times 10^4$ photons/sr and $4.3 \times 10^4$ photons/sr for scattering with the chirped and transform-limited laser pulses, respectively.